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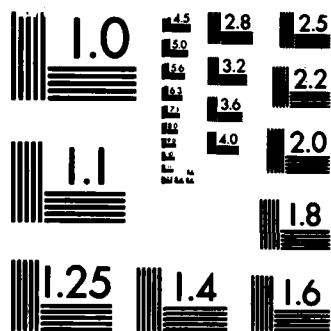
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SUP-NORM ESTIMATES IN GLIMM'S METHOD

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ABSTRACT

We give a proof that Total Variation $\{u_0(\cdot)\} \ll 1$ can be replaced by $\text{Sup}\{u_0(\cdot)\} \ll 1$ in Glimm's method whenever a coordinate system of Riemann invariants is present. The argument is somewhat simpler but in the same spirit as that given by Glimm in his celebrated paper of 1965.

AMS (MOS) Subject Classifications: 65M10, 76N99, 35L65, 35L67

Key Words: Riemann problem, Random choice method, Stability,
Conservation Laws, Cauchy problem

Work Unit Number 1 (Applied Analysis)

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SIGNIFICANCE AND EXPLANATION

A system of two conservation laws in one dimension is a set of first order nonlinear partial differential equations of the form ^{a certain}

$$(1) \quad u_t + f(u,v)_x = 0,$$

$$v_t + g(u,v)_x = 0,$$

where (u,v) is a vector function of (x,t) , $x \in R$, $t \geq 0$. The Cauchy problem asks for a solution of (1) given the ^{epsilon}initial values of u and v at time $t = 0$. Equations of type (1) arise, for example, in gas dynamics where they express the conservation of quantities like mass, momentum and energy, when diffusion is neglected. Typically, smooth solutions of (1) cannot be found. This is due to the formation of shock waves. Shock waves are the mechanism by which entropy is dissipated in solutions of (1). This paper gives a proof that solutions exist even after shock waves form, so long as the amplitude of the waves are not too great initially.

Keywords: Riemann invariants; Conservation laws; stability

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SUP-NORM ESTIMATES IN GLIMM'S METHOD

Blake Temple

We consider the Cauchy problem

$$u_t + F(u)_x = 0 \quad (1)$$

$$u(x,0) = u_0(x), \quad (2)$$

where (1) denotes a strictly hyperbolic system of two conservation laws, $u = (u_1, u_2)$, $F = (f, g)$. Let λ_p, R_p , $p = 1, 2$ be the eigenvalues and corresponding eigenvector fields associated with the matrix ∇F , $\lambda_1 < \lambda_2$. Assume that U is a neighborhood of a state \bar{u} in u -space in which each characteristic field is either genuinely nonlinear ($\nabla \lambda_p \cdot R_p > 0$) or else linearly degenerate ($\nabla \lambda_p \cdot R_p \equiv 0$), and such that $\lambda_1(u) < \lambda_2(v)$ for all $u, v \in U$ [4]. Without loss of generality, assume $\bar{u} = 0$. In this note we give a simplified proof of the following result which is contained in the results of Glimm [2] and which is required for the proof in [12]. (A stronger result also follows from the analysis in [3] which, however, involves the theory of approximate characteristics and is much more technical.) Let $u(x, t)$ denote a weak solution of (1), (2) which is a limit of approximate solutions generated by the random choice method of Glimm.

Theorem 1: For every $V_0 > 0$ there exists a small constant $\delta > 0$ and a large constant $G > 0$ such that, if

$$TV\{u_0(\cdot)\} < V_0, \quad (3)$$

and

$$\|u_0(\cdot)\|_S < \delta, \quad (4)$$

then

$$\|u(\cdot, t)\|_S < G\|u_0(\cdot)\|_S, \quad (5)$$

for all $t > 0$. Here

$$\|u_0(\cdot)\|_S \equiv \text{Sup}\{u_0(\cdot)\}. \quad (6)$$

It suffices to verify Theorem 1 for any approximate solution u^h generated by the random choice method. Recall that there exists a coordinate system of Riemann invariants for (1) in a neighborhood of $u = 0$. (Indeed, Theorem 1 and the proof to follow apply to any system satisfying the above assumptions and for which there exists a coordinate system of Riemann invariants [2].) Moreover, the Riemann problem is uniquely solvable in a neighborhood of $u = 0$ by the method of Lax [4]. Such a solution consists of a 1-wave γ^1 followed by a 2-wave γ^2 each of which is either a shock wave or a rarefaction wave. Assume that U is a neighborhood of $u = 0$ satisfying all of the above conditions. Define the strength of a wave $|\gamma^p|$ to be the absolute value of the change in the opposite Riemann invariant between the left and right states of the wave. Finally, in order to set notation, we briefly review the construction of the random choice method approximate u^h .

Let h be a mesh length in x , and let

$$k = Ch$$

be the corresponding mesh length in t , $C > \sup_{p, u \in U} \{|\lambda_p(u)|\}$. For $i, j \in \mathbb{Z}$, $j > 0$, let $x_i \equiv ih$, $t_j \equiv jk$. Let $\{a_j\}$ be a sample sequence, $a_j \equiv \{a_j\}_{j=1}^\infty$, $0 < a_j < 1$. For given initial data $u_0(\cdot) \subset U$, define the random choice method approximate solution $u^h(x, t) \equiv u^h(x, t; a_j)$ by induction on j as follows: First, for $x_i < x < x_{i+1}$, define

$$u^h(x, 0) \equiv u_0(x) = u_0(x_i + \frac{h}{2}).$$

Next, assume for induction that $u^h(x, t)$ has been defined for $t < t_j$. Define

$$u^h(x, t_j) \equiv u^h(x_i + a_j h, t_j^-),$$

and for $t_j < t < t_{j+1}$, define $u^h(x, t)$ to be the solution of the Riemann problem posed in (3.3) at time t_j . By (3.1), u^h is well defined so long as $u^h(x, t_j) \subset U$ for all t_j . Let γ_{ij}^p denote the p -wave that appears in the solution of the Riemann problem that is posed at (x_i, t_j) in the approximate solution u^h . Recall that the quadratic functional associated with u^h is defined by

$$Q(t) \equiv \sum |\gamma_{ij}^p| |\gamma_{i,j}^{p'}| \quad (7)$$

where the sum is over all waves that approach at time t_j , $t_j < t < t_{j+1}$. Let Δ_{ij} denote the interaction diamond centered at (x_i, t_j) , and let D_{ij} denote the products of

approaching waves that enter Δ_{ij} [12]. We use the following notation:

$$V_j \equiv \sum_{p,i} |\gamma_{ij}^p|, \quad (8)$$

$$Q_j \equiv Q(t_j) = Q(t_j+), \quad (9)$$

$$D_j \equiv \sum_i D_{ij} \quad (10)$$

$$F_j \equiv V_j + Q_j \quad (11)$$

$$S_j \equiv \|u^h(\cdot, t_j)\|_S. \quad (12)$$

Note that V_j estimates the total variation of $u^h(\cdot, t_j)$ and that (7), (9) give immediately that

$$Q_j < V_j^2. \quad (13)$$

We show that Theorem 1 is a consequence of the following lemma which is a restatement of results in [2]:

Lemma 1: There exists a constant $G_0 > 1$ depending only on F such that, if $u^h(x, t) \in U$ for all $t < t_j$, then

$$V_{j+1} - V_j < G_0 S_j D_j, \quad (14)$$

$$S_{j+1} - S_j < G_0 S_j D_j, \quad (15)$$

$$Q_{j+1} - Q_j < (G_0 S_j V_j - 1) D_j. \quad (16)$$

Proof of Theorem 1: Fix $V_0 > 1$. Choose

$$S_0 < (8G_0^2 V_0^2 e^{2G_0 V_0^2 - 1})^{-1} = \delta, \quad (17)$$

where G_0 is large enough so that

$$\{u : \|u\| < G_0^{-1}\} \subseteq U. \quad (18)$$

We show by induction that (17) implies

$$S_j < e^{2G_0 V_0^2} S_0, \quad (19)$$

and

$$G_0 S_j V_j < \frac{1}{4} \quad (20)$$

for all $j > 0$. Note that (19) gives (5) with $G = e^{2G_0 v_0^2}$. Also note that (10) and (11) imply

$$G_0 S_j < \frac{1}{4}. \quad (21)$$

Thus, since (8) gives

$$Q_{j+1} - Q_j < (G_0 S_j v_j - 1) D_j, \quad (22)$$

$$F_{j+1} - F_j < (G_0 S_j + G_0 S_j v_j - 1) D_j, \quad (23)$$

estimates (19) and (20) also imply that

$$Q_{j+1} - Q_j < -\frac{1}{2} D_j, \quad (24)$$

$$F_{j+1} - F_j < -\frac{1}{2} D_j. \quad (25)$$

We now verify (19) and (20) by induction. The idea here is that (20) guarantees that both $\{Q_j\}$ and $\{F_j\}$ are decreasing. The decreasing of Q_j controls S_j at the induction step, while the decreasing of $\{F_j\}$ maintains (20) at the induction step, since then $v_j < F_j < F_0$.

First, when $j = 0$, (10) and (11) follow from (17). So assume (10), (11) hold for $j' < j$. We verify (10), (11) for $j' = j + 1$. By (24) and (15),

$$S_{k+1} - S_k < 2G_0 S_k [Q_k - Q_{k+1}],$$

or

$$S_{k+1} < (1 + 2G_0 [Q_k - Q_{k+1}]) S_k, \quad (26)$$

for $k < j + 1$. Thus by (26)

$$S_{j+1} < \prod_{k=0}^j (1 + 2G_0 [Q_k - Q_{k+1}]) S_0. \quad (27)$$

But one can easily verify that the maximum of

$$\prod_{k=0}^j (1 + a_k)$$

over all nonnegative sequences $\{a_k\}_{k=0}^j$ satisfying $\sum_{k=0}^j a_k < M$ is attained when $a_k = \frac{M}{j+1}$ for all k . Thus by (13) and (24),

$$\sum_{k=0}^j [Q_k - Q_{k+1}] < Q_0 < v_0^2, \quad (28)$$

so

$$\prod_{k=0}^j \{1 + 2G_0(Q_k - Q_{k+1})\} < \left\{1 + \frac{2G_0 v_0^2}{j+1}\right\}^{j+1} < e^{2G_0 v_0^2}. \quad (29)$$

Therefore by (27),

$$s_{j+1} < e^{2G_0 v_0^2} s_0,$$

which verifies (19) at $j + 1$. Moreover, (25) implies

$$v_{j+1} < F_{j+1} < F_0 < 2v_0^2.$$

Thus by (17),

$$G_0 s_{j+1} v_{j+1} < 2G_0 e^{2G_0 v_0^2} v_0^2 < \frac{1}{4},$$

which verifies (20) at $j + 1$. This completes the proof of Theorem 1.

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